Enrollment No: ______________________ **Exam Seat No:**______________________

C. U. SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Engineering Mathematics-IV

Subject Code: 4TE04EMT1 **Branch: B.Tech (All)**

Semester: 4 **Date: 19/09/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Relation between E and Δ

(a). $\Delta = E - 1$ (b). $\Delta = E + 1$ (c). $\Delta = 1 - E^{-1}$ (d). All of these

b) $curl(grad\ \phi) =$ _______.

(a). 2 (b). 1 (c). 0 (d). -1

c) The fixed points of the mapping $w = \frac{-z+1}{z-1}$

(a). 2, 1 (b). 1, -1 (c). 1, -2 (d). 2, -2

d) Let $f(x, y, z) = c$ represent the equation of a surface, Unit normal vector is___

(a). $\frac{\text{grad } f}{|\text{grad } f|}$ (b). grad f (c). div(grad f) (d). curl(grad f)

e) if $\bar{A}(t) = 3t^2\hat{\imath} + 2t\hat{\jmath} + 4t^3\hat{k}, \int_{t=1}^{t=2} \bar{A}(t)dt\ equal\ to$

(a). $7\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ (b). $7\hat{\imath} + 3\hat{\jmath} + 15\hat{k}$

(c). $-7\hat{\imath} - 3\hat{\jmath} + 15\hat{k}$ (d).None of these

f) If $f(z) = u(x, y) + i\ v(x, y)$ is analytic then $f'(z) =$ ______.

(a). $u_x + i\,v_x$ (b). $u_x - i\,v_x$ (c). $u_y + i\,v_x$ (d). $u_x + i\,v_y$

g) The value of $\int_c \frac{dz}{z-9}$. $C: |z| = 5$

(a). $2\pi i$ (b). $-2\pi i$ (c). $4\pi i$ (d). 0

h) In Gauss- Jordan method coefficient matrix reduce into

(a). Upper triangular matrix (b). Lower triangular matrix
(c). Unit Matrix (d). Diagonal Matrix

i) Which method is known as Self correction method?

(a). Gauss- elimination method (b). Gauss- Jordan method (c). Gauss Seidel method (d). Gauss-Jacobi method

j) If $f(x)$ is even then

(a). $B(\lambda) = 0$ (b). $A(\lambda) = 0$ (c). Both a and b (d). None of these

k) $E^5 f(x) =$ _____

(a). $5f(x+h)$ (b). $f(x+5h)$ (c). $f(x-5h)$ (d). None of these

l) Which one of the following method is more rapid in convergence than Gauss-Jacobi method

(a). Gauss- elimination method (b). Gauss- Jordan method

(c). Gauss Seidel method (d). None of these

m) Putting $n = 3$ in Newton- cote's formulae, we get _____

(a). Trepezoidal Formula (b). Simpson's $\frac{1}{3}$ rule

(c). Simpson's $\frac{3}{8}$ rule (d). None of these

n) Write Heat Equation.

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions **(14)**

a) Given $\vec{u} = x\,\hat{\imath} + (2x^2 - y^2)\,\hat{\jmath} + z^3\,\hat{k}$ and (06)

$v = y + xz + xz^2$ then find $\nabla \cdot \vec{u}$ and $\nabla \cdot v$ and $\nabla \times \vec{u}$.

b) Show that : **a.** $\nabla^2 \left(\frac{1}{r}\right) = 0$ **b.** $\nabla^2 (r^m) = m(m+1)r^{m-2}$ (04)

c) If $\vec{A}$ and $\vec{B}$ are irrotational, show that $\vec{A} \times \vec{B}$ is solenoidal. (04)

Q-3 Attempt all questions **(14)**

a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied. (05)

b) Evaluate $\oint_c |z|^2 dz$, around the square with vertices (0,0), (1,0), (1,1), (0,1). (05)

c) Determine the mobius transformation that maps $z_1 = 2, z_2 = 3, z_3 = \infty$ onto $w_1 = -3, w_2 = -5, w_3 = 3$ respectively. What are the invariant points of the transformation? (04)

Q-4 Attempt all questions **(14)**

a) Applying Green's Theorem, Evaluate $\oint_C [(y - \sin x)dx + \cos x\ dy]$, where C is the plane triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$. (07)

b) Evaluate $\iint_S (\nabla \times \vec{F}) \cdot d\,\vec{S}$ taken over the portion of the surface $x^2 + y^2 - 2ax + az = 0$ and the bounding curve in the plane $z = 0$ and $\vec{F} = (y^2 + z^2 - x^2)\hat{\imath} + (z^2 + x^2 - y^2)\hat{\jmath} + (x^2 + y^2 - z^2)\hat{k}$. (05)

c) State Stoke's theorem. (02)

Q-5 Attempt all questions **(14)**

a) Determine the analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$ (05)

b) Solve by using Gauss-elimination method (05)

$5x - 2y + 3z = 18, \quad x + 7y - 3z = -22, \quad 2x - y + 6z = 22$

c) Use Lagrange's interpolation formula to find the value of y when $x = 10$. (04)

x	0	2	3	6
y	648	704	729	792

Q-6 **Attempt all questions** **(14)**

a) Sove the following system by using Gauss-Seidel method (05)

$27x + 6y - z = 85, \quad 6x + 5y + 2z = 72, \quad x + y + 54z = 110$

b) Using Taylor series method, find $y(1.1)$ correct to four decimal places, given that (05)

$$\frac{dy}{dx} = xy^{\frac{1}{3}}, y(1) = 1.$$

c) Construct Newton's forward interpolation polynomial for the following data: (04)

x	4	6	8	10
y	1	3	8	16

Use it find the value of y for $x = 5$.

Q-7 **Attempt all questions** **(14)**

a) Given $\sin 45° = 0.7071$,$\sin 50° = 0.7660$,$\sin 55° = 0.8192$, $\sin 60° = 0.8660$ then find $\sin 52°$ using Newton's forward Interpolation formula. (05)

b) Compute f8 from the following value of Newton's Divided difference formula (05)

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

c) Find the fourier cosine and sine transforms of the function (04)

$$f(x) = \begin{cases} k & if \;\; 0 < x < a \\ 0 & if \;\; x > a \end{cases}$$

Q-8 **Attempt all questions** **(14)**

a) Find the fourier transform of $e^{-(a^2x^2)}, a > 0$ and deduce that (05)

$$F\left(e^{-\frac{\lambda^2}{2}}\right) = e^{-\frac{\lambda^2}{2}}.$$

b) Using Cauchy's integral formula, evaluate: $\oint_C \frac{\sin^2 z}{\left(z-\frac{\pi}{6}\right)^3} dz\,,\; C: |z| = 1$ (05)

c) If $y_0 = 3, y_1 = 12, y_2 = 81, y_3 = 2000,$ and $y_4 = 100$ then find $\Delta^4 y_0$. Also write Newton forward interpolation formula. (04)